

① Discrete random variable:

What is it? Finite number of values $a \leq z \leq b$

② Notation

$$f(Z=z) = [z]$$

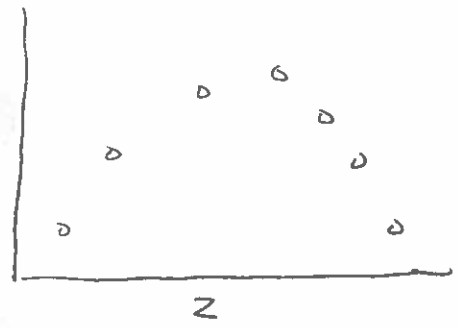
Support all values of z for which $[z] > 0$ and defined
Probability mass function

③ Probability mass function: not necessarily integer
A function for which: 2 things

$$[z] \geq 0$$

$$\sum_{z \in S} z = 1$$

note scale \rightarrow Probability of z



④ Example using Species Richness on many plots:

$$[z|k] = \frac{\lambda^z e^{-\lambda}}{z!}$$

Moment generating function

$$M_j = \sum_{z \in S} (z-c)^j [z]$$

Moments

approximated by many random draws from the distribution of z using $\frac{1}{n} \sum_{i=1}^n z_i$

$c=0$
 $j=1$

$$E(z) = \mu = \sum_{z \in S} z [z]$$

Expected value or mean
First moment

$c=\mu$
 $j=2$

$$E(z-\mu)^2 = \sigma^2 = \sum_{z \in S} (z-\mu)^2 [z]$$

Second central moment
variance

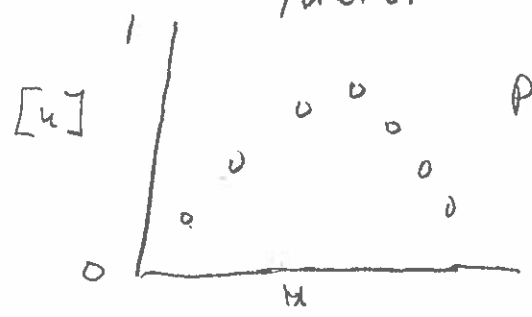
$$\frac{1}{n} \sum_{i=1}^n (z_i - \mu)^2$$

Other moments

$j=3$ skewness

$j=4$ kurtosis "fatness of tails"

Probability mass function



Cumulative distribution function

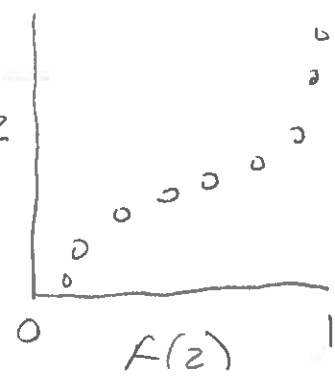
$$F(z) = \sum_{u \leq z} [u]$$

$F(z) = 1$
Pr $u \leq z$



when is it useful?

Quantile function



① Continuous random variable
 $a < z < b$, infinite number of values

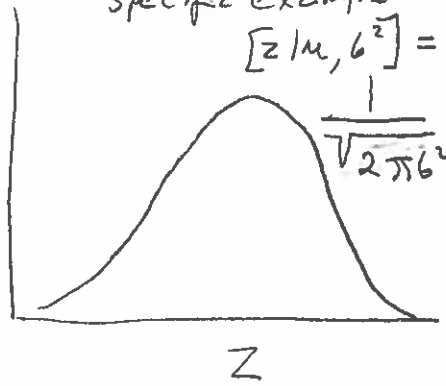
Requirements for probability density function

- ① $[z] \geq 0$
- ② $\int_a^b [z] dz = \Pr(a \leq z \leq b)$
- ③ $\int_{-\infty}^{\infty} [z] dz = 1$

$[z]$
 probability density of z

support $[z] \geq 0$ and defined

specific example
 $[z|m, \sigma^2] = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(z-m)^2}{2\sigma^2}}$



Moment generating function

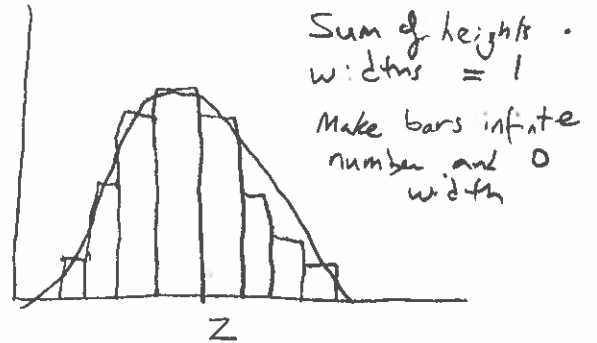
$$m_j = \int_{-\infty}^{\infty} (z-c)^j [z] dz$$

First moment:
 Expected value or mean

$c=0$
 $j=1$
 $m_1 = E(z) = \mu = \int_{-\infty}^{\infty} z [z] dz$
 approximated as $\frac{1}{n} \sum_{i=1}^n z_i$

Second central moment:

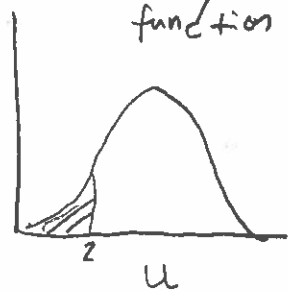
$c=\mu$
 $j=2$
 $m_2 = E(z-\mu)^2 = \sigma^2 = \int_{-\infty}^{\infty} (z-\mu)^2 [z] dz$
 approximated as $\frac{1}{n} \sum_{i=1}^n (z_i - \mu)^2$



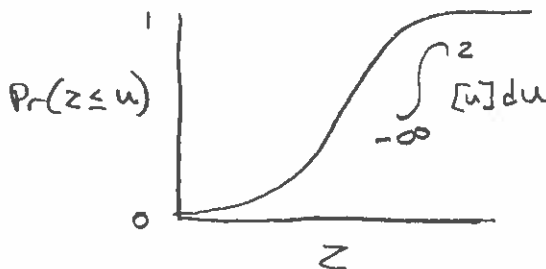
Probability density are values on the continuous curve such that the area under the curve = 1

Scaling of y axis

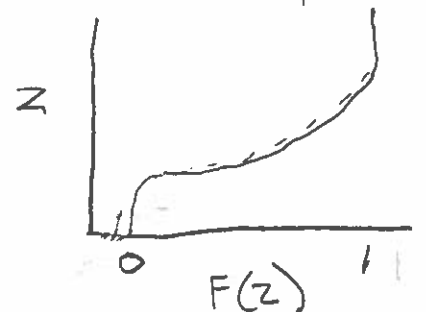
Probability density function



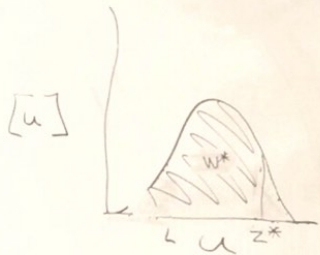
$F(z) =$ Cumulative distribution function =



$F'(z) =$ quantile function

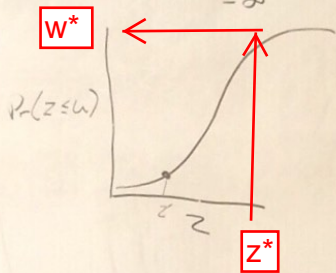


Probability density function



Cumulative distribution function

$$F(z) = \int_{-\infty}^z [u] du$$



Quantile function

$$F^{-1}(z)$$

